On the Relationship Between Capacity and Distance in an Underwater Acoustic Communication Channel

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Motivation

- Underwater acoustic networks:
  - system design and optimization
  - fundamental limitations and system capacity.
- Fact: acoustic bandwidth depends on distance (because acoustic path loss increases with frequency).
- Ex. 1 kHz over 100 km, 10 kHz over few km.
- More bandwidth is available if information is transmitted over multiple short hops than over one long hop.
- Q: What exactly is the dependence between bandwidth (link capacity) and distance?
Outline

• Attenuation and noise in an acoustic channel  
  (first approximation: single-path, time-invariant model)
• SNR and optimal frequency
• Bandwidth, capacity, power needed to achieve a pre-specified SNR as functions of distance
• Numerical example
• Closed form approximations
Attenuation and Noise

\[ A(l, f) = l^k a(f)^l \]

\[ N(f) \sim N f^{-\eta} \]

\[ 10 \log a(f) = 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f} + 2.75 \cdot 10^{-4} f^2 + 0.003 \]

\[ H(l, f) = \sum_{p=0}^{P-1} \Gamma_p \sqrt[2]{A(l_p, f) e^{-j2\pi f \tau_p}} \]

\[ A(l, f) \to 1/|H(l, f)|^2 \]

\[ 10 \log N(f) \approx 50 - 18 \log f \]

\[ 10 \log N_t(f) = 17 - 30 \log f \]

\[ 10 \log N_s(f) = 40 + 20(s - 0.5) + 26 \log f - 60 \log(f + 0.03) \]

\[ 10 \log N_w(f) = 50 + 7.5w^{1/2} + 20 \log f - 40 \log(f + 0.4) \]

\[ 10 \log N_{th}(f) = -15 + 20 \log f \]
Narrowband SNR and Optimal Frequency

\[
SNR(l, f) = \frac{P/A(l,f)}{N(f)\Delta f} \sim \frac{1}{A(l,f)N(f)}
\]

\[
f_o(l) = \arg\max_f SNR(l, f)
\]

\[k = 1.5, s = 0.5, w = 0\]
Bandwidth: A Heuristic Definition

• Define 3 dB bandwidth as that range of frequencies for which
  \( SNR(l, f) > \frac{SNR(l, f_o(l))}{2} \) (or use some other definition) ⇒

• \( B(l) = [f_{\text{min}}(l), f_{\text{max}}(l)] \): bandwidth around \( f_o(l) \), e.g. \( B_3(l) \).

• For given \( B(l) \), set tx power \( P(l) \) to achieve a pre-specified SNR:
  \[
  SNR(l, B(l)) = \frac{\int_{B(l)} S_l(f) A^{-1}(l, f) df}{\int_{B(l)} N(f) df} \geq SNR_0
  \]

• Equal energy distribution:
  \[
  P(l) = \int_{B(l)} S_l(f) df = S_l B(l)
  \]

• Tx power:
  \[
  P_3(l) = SNR_0 \cdot B_3(l) \frac{\int_{B_3(l)} N(f) df}{\int_{B_3(l)} A^{-1}(l, f) df}
  \]
Optimal Energy Allocation

- Capacity (channel is Gaussian, but not white):
  \[ C(l) = \int_{B(l)} \log_2 \left[ 1 + \frac{S_l(f)}{A(l, f)N(f)} \right] df \]
  \[ S_l(f) + A(l, f)N(f) = K_l \]

- Constant $K_l$ to be determined from the constraint on fixed tx power– minimum needed to satisfy
  \[ SNR(l, B(l)) = \frac{\int_{B(l)} S_l(f) A^{-1}(l, f) df}{\int_{B(l)} N(f) df} = K_l \frac{\int_{B(l)} A^{-1}(l, f) df}{\int_{B(l)} N(f) df} - 1 \geq SNR_0 \]

- Tx power:
  \[ P(l) = \int_{B(l)} S_l(f) df = K_l B(l) - \int_{B(l)} A(l, f)N(f) df \]
Bandwidth: Capacity Maximizing Definition

\[
C(l) = \int_{B(l)} \log_2 \left[ \frac{K_l}{A(l, f)N(f)} \right] df, \quad C_3(l) = \int_{B_3(l)} \log_2 \left[ 1 + \frac{P_3(l)/B_3(l)}{A(l, f)N(f)} \right] df
\]
Numerical Example: SNRo=20dB

3 dB definition

\[
\hat{B}_3(l) = b_3 l^{-\beta_3}, \quad \hat{C}_3(l) = c_3 l^{-\gamma_3}, \quad \hat{P}_3(l) = p_3 l^{\pi_3},
\]

capacity-based definition

\[
\hat{B}(l) = b_o l^{-\beta_o}, \quad \hat{C}(l) = c_o l^{-\gamma_o}, \quad \hat{P}(l) = p_o l^{\pi_o}
\]
Curve Fitting

- Least-squares approximation by a first-order polynomial on a logarithmic scale
- Coefficients of the semi-analytical solution for bandwidth, capacity, and transmission power at SNR0=20 dB:

<table>
<thead>
<tr>
<th></th>
<th>heuristic 3 dB definition</th>
<th>optimal definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>bandwidth</td>
<td>$b_3=14.39$ dB re kHz</td>
<td>$b_0=19.76$ dB re kHz</td>
</tr>
<tr>
<td></td>
<td>$\beta_3=-0.55$ dB re kHz/km</td>
<td>$\beta_0=-0.59$ dB re kHz/km</td>
</tr>
<tr>
<td>capacity</td>
<td>$c_3=22.68$ dB re kbps</td>
<td>$c_0=28.76$ dB re kbps</td>
</tr>
<tr>
<td></td>
<td>$\gamma_3=-0.55$ dB re kbps/km</td>
<td>$\gamma_0=-0.59$ dB re kbps/km</td>
</tr>
<tr>
<td>power</td>
<td>$p_3=106.78$ dB re $\mu$Pa</td>
<td>$p_0=127.25$ dB re $\mu$Pa</td>
</tr>
<tr>
<td></td>
<td>$\pi_3=2.22$ dB re $\mu$Pa/km</td>
<td>$\pi_0=2.07$ dB re $\mu$Pa/km</td>
</tr>
</tbody>
</table>
Summary of Results for Varying SNR: Bandwidth Efficiency

\[ \frac{C(l)}{B(l)} \text{ vs. } P(l) \quad \frac{C(l)}{B(l)} \text{ vs. } SNR_0 \quad \frac{C(l)}{B(l)} \text{ vs. } \frac{E_b}{N_0} \]

\[ \left( \frac{C}{B} \right)_{AWGN} = \log_2(1 + SNR_0) \]

\[ \frac{E_b}{N_0} = \frac{2(C/B)_{AWGN} - 1}{(C/B)_{AWGN}} \]

equivalent AWGN channel: \[ N_0(l) = \frac{1}{B(l)} \int_{B(l)} N(f) df, \quad E_b(l) = \frac{1}{C(l)} \int_{B(l)} S_l(f) A^{-1}(l, f) df \Rightarrow \]

\[ E_b/N_0 = SNR_0(l, B(l))B(l)/C(l) \]
Conclusion

• Acoustic bandwidth decreases with transmission distance ⇒ Relaying information over multiple hops can increase throughput (and decrease energy consumption).

• Practical network design:
  – How many relays to use? Where to place them? How to optimally allocate resources?
  – What is the overall throughput improvement? What is the network capacity?

• Starting point: semi-analytical solution for bandwidth, capacity, power, as functions of distance (for a very simple model).

• Future work:
  – extend results to time-varying multipath channel (model?)
  – use link-level results to assess network capacity.